

A theoretically-explained new-variant of Modified-Newtonian-Dynamics (MOND)

R. C. Gupta ¹ and Anirudh Pradhan ²

¹ GLA Institute of Technology and Management (GLAITM),
Mathura-281 406, India

E-mail: rcg_iet@hotmail.com, rcgupta@glaitm.org

² Department of Mathematics, Hindu P. G. College,
Zamania-232 331, Ghazipur, India

E-mail: acpradhan@yahoo.com, apradhan@imsc.res.in

Abstract

It is surprising that we hardly know only 5% of the universe. Rest of the universe is made up of 70% of dark-energy and 25% of dark-matter. Dark-energy is responsible for acceleration of the expanding universe; whereas dark-matter is said to be necessary as extra-mass of bizarre-properties to explain the anomalous rotational-velocity of galaxy. Though the existence of dark-energy has gradually been accepted in scientific community, but the candidates for dark-matter have not been found as yet and are too crazy to be accepted. Thus, it is obvious to look for an alternative theory in place of dark-matter. Israel-scientist M. Milgrom has suggested a 'Modified Newtonian Dynamics (MOND)' which appears to be highly successful for explaining the anomalous rotational-velocity. But unfortunately MOND lacks theoretical support. The MOND, in-fact, is (empirical) modification of Newtonian-Dynamics through modification in the kinematical acceleration term 'a' (which is normally taken as $a = \frac{v^2}{r}$) as effective kinematic acceleration $a_{effective} = a\mu(\frac{a}{a_0})$, wherein the μ -function is 1 for usual-values of accelerations but equals to $\frac{a}{a_0} (\ll 1)$ if the acceleration 'a' is extremely-low lower than a critical value $a_0(10^{-10} m/s^2)$. In the present paper, a novel variant of MOND is proposed with theoretical backing; wherein with the consideration of universe's acceleration a_d due to dark-energy, a new type of μ -function on theoretical-basis emerges out leading to $a_{effective} = a(1 - K\frac{a_d}{a})$. The proposed theoretical-MOND model too is able to fairly explain qualitatively the more-or-less 'flat' velocity-curve of galaxy-rotation, and is also able to predict a dip (minimum) on the curve.

Key words : Cosmology, Dark Matter, MOND, Dark Energy

PACS: 98.80.-k, 95.36.+x, 95.35.+d

1 Introduction

How much matter and energy are there in the universe ? It is now well established that universe-expansion began with a big-bang [1]. The ultimate fate of the universe depends on the universe's matter & energy density (ρ) as compared to a certain value called critical-density (ρ_c). If $\rho > \rho_c$, the universe is said to be 'closed'; and its expansion will slow down (decelerate) and start contracting leading finally to a big-crunch (meaning hot-death of the universe). If $\rho < \rho_c$, the universe is said to be 'open'; and will expand forever even much faster (leading the universe to cold-death). If $\rho = \rho_c$, the universe is said to be 'flat'; and will continue to expand but not that-fast to lead to cold-death soon. The ratio $k = \frac{\rho}{\rho_c}$, determine that whether the universe is closed ($k > 1$), open ($k < 1$) or flat ($k = 1$). It has been estimated that the universe would have collapsed (to hot-death) much sooner than the present-age of the universe if $k > 1$; and it would have cooled down (to cold-death) much earlier than the present-age of the universe if $k < 1$. The present-age (14 billion years) constraint of the universe, compel the scientists to believe that $k = 1$, i.e., the universe must be flat [1].

Once agreed-upon that the universe density $\rho = \rho_c$, the next question arises that 'what is the universe made of'? Estimation of visible-type matter like galaxies, stars, planets etc. hardly leads only to about 2% of ρ_c ; and when other all such things like inter-galactic gases, black-hole, white-dwarf, neutron-stars etc. are also included, the estimate hardly reaches a mere 5% of ρ_c . What is then 95% of the remaining-part ? It seems invisible and unknown, hence thought as dark constituent(s). Scientists have, presently, estimated that the major-chunk of the universe [2, 3] is repulsive-gravity type dark-energy (about 70%) causing the universe's accelerated expansion, and the rest is non-baryonic invisible but gravitating dark-matter (about 25%) causing anomalous high rotational-speed of galaxies.

Recognition of dark-energy mainly through Supernovae (SNe Ia) observations [4-9], galaxy cluster measurements [10] and cosmic microwave background (CMB and WMAP) data [11, 12] is comparatively a recent affair; the possibility (necessity) of dark-matter was anticipated quite-early (in 1935 by Fritz Zwicky), but the work of Fritz Zwicky [13, 14] was largely ignored. Much later (1960s to 1980s), it is the female-scientist Veera Rubins concentrated efforts [15, 16] which made the male-scientists to finally take the possibility of dark-matter seriously. Now, based on the rotational-velocity curves of galaxies and galaxy-clusters, the scientific community generally believe the necessity of dark-matter. But the next question is rather yet unanswered that 'what makes the dark matter'? Dark matter seems to be non-baryonic. Several schools of thought have emerged [3], none of these very satisfactory; can be grouped in two categories: (i) Hot dark matter and (ii) Cold dark matter. Neutrino is the main candidate for hot dark matter, but unable to take-up the full account. Exotic sub-particles, the candidates for the cold dark matter are: 'weakly interactive

massive particles’ (WIMPs) and ‘massive astrophysical compact halo objects’ (MACHOs); WIMPs include exotic extremely-long ‘axion’ and new-breed of particles named ‘s-particles’(super-partner of particles, a possibility based on recent super-symmetry theory).

Though there seems a need of dark-matter inside the galaxy to explain the anomaly of galaxy’s rotational-speed (higher rotational-speed requires more mass inside), but the candidates of dark-matter are so strange to be believed (as if, dark-matter is castle-in-air). We wish if there is any way out, to explain the anomaly of rotational-speed of galaxies, without the need of dark-matter. In fact *that is what* the Israel-scientist Mordehai Milgrom did and proposed a ‘Modified Newtonian Dynamics (MOND)’. We are now left with two alternatives/options: (1) Believe in the existence of the crazy dark-matter, and try to find out what (candidate) makes the dark-matter, OR (2) Use Milgrom’s MOND theory (or its variant) which eliminates the need of dark-matter altogether.

There would appear some confusions about the ‘acceleration symbols and term(s)’ and problem in the understanding the Milgrom’s MOND theory (section-3) and its new variant proposed in this paper (section-4), unless the term ‘acceleration’ is re-examined, re-defined and clarified as follows (in section-2).

2 Acceleration Re-defined

It is said that ‘*everything thing on earth and beyond, is governed by law of mechanics*’. Mechanics is subdivided into ‘Statics’ and ‘Dynamics’. Dynamics is categorized further in two categories: ‘Kinematics’ and ‘Kinetics’. The ‘kinematics’ is that part of dynamics wherein mass is of no concern, such as displacement and velocity; whereas ‘kinetic’ refers to the dynamics which depend on mass, such as force and moment-of-inertia. It has not been appreciated earlier in physics that ‘acceleration’, in fact belongs to both i.e., there are two types of acceleration viz., ‘kinematic acceleration’ and ‘kinetic acceleration’. If a particle is moving in circular-path, then due to kinematical change in velocity towards center, it would have a ‘kinematic-acceleration a ’ towards center commonly referred as centripetal-acceleration $\frac{v^2}{r}$, which is also sometimes referred as pseudo centrifugal-acceleration. On the other hand the gravitational- acceleration, say on a star in spiral galaxy-arm, can be found using Newton’s second law as ‘force divided by star-mass m ’ as $a_n = \frac{GM}{r^2}$, which is in fact ‘kinetic-acceleration a_n ’ (the subscript n in a_n refers to N of Newton’s law). Normally, both of these ‘kinetic’ and ‘kinematic’ accelerations are equal hence considered as same, that is why there has been no need for any differentiation. But, time has come to differentiate between ‘kinetic’ and ‘kinematic’ accelerations, for better understanding of Milgrom’s MOND and herewith proposed MOND-variant wherein there would be some changes in the kinematic-acceleration; the new effective kinematic-acceleration $\mathbf{a_{effective}} = \mathbf{a}$ (μ **function**). The μ -function is taken to be 1 for usually encountered acceleration, but less than 1 if the accel-

eration is extremely small. In all the situations, the kinetic-acceleration equals the effective-kinematic-acceleration, mathematically speaking, $\mathbf{a_n} = \mathbf{a_{effective}}$; the right-hand-side will be equal to kinematic acceleration ‘ a ’ or less than ‘ a ’ depending whether the value of usual kinematic acceleration ‘ a ’ is greater than a critical value (a_0) or less than that (See section-3 of MOND wherein it is described in detail).



Figure 1: A Typical Spiral Galaxy

3 Modified Newtonian Dynamics (MOND)

Modified Newtonian Dynamics (briefly abbreviated as MOND) was proposed by Milgrom [17, 18] as modification in ‘kinematic acceleration’, at extremely low acceleration, to explain the galaxy-rotation problem. MOND eliminates the need of dark-matter.

Detailed observations of rotational speed of galaxies (Fig.1), in 1980, made it clear that galaxies do not exhibit the same pattern of decreasing orbital velocity with increasing distance from the center-of-mass as observed in the Solar-system.

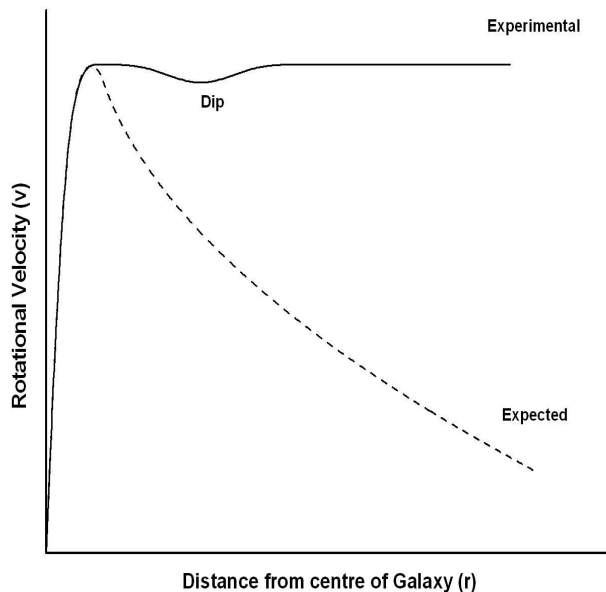


Figure 2: Rotational velocity versus distance from Galaxy-centre: expected curve based on Newtonian dynamics and experimental ‘flat’ curve with a dip

A spiral galaxy (Fig.1) consists of a bulge of stars at the center with a vast number of stars orbiting around the central solid-type big-lump. If the orbits of the galaxy’s stars were solely governed by the central-gravitational force, it was expected that the stars at the outer edge of the galaxy would have a much lower orbital-velocity than that of those near the middle of the galaxy. In the observed galaxies, this velocity pattern is not noticed; stars near the outer edge were found to be orbiting at about the same-speed as the inner stars near the middle (Fig.2). This “flattening of galaxy’s rotational-curve’ requires invisible dark-matter within the galaxy, or necessitates the use of Modified Newtonian Dynamics (MOND).

The MOND theory, in fact, modifies the ‘kinematic-acceleration’ with a μ -function as follows, specially when acceleration is extremely small $a \ll a_0$ ($a_0 \approx 10^{-10} m/s^2$). According to MOND, the modified ‘effective kinematic-acceleration’ is given as

$$a_{effective} = a \mu\left(\frac{a}{a_0}\right), \quad (1)$$

$\mu(\frac{a}{a_0})$ is termed as μ -function and $a_0 \approx 10^{-10}m/s^2$. The function $\mu(\frac{a}{a_0})$ turns out to be as follows.

$$\begin{aligned}\mu\left(\frac{a}{a_0}\right) &= 1 && \text{for } a \gg a_0, \\ &= \frac{a}{a_0} && \text{for } a \ll a_0.\end{aligned}\tag{2}$$

Now applying MOND to the gravitational attraction-force between a star to the central galaxy core (of mass M),

$$a_n = a_{effective}$$

i.e.

$$\frac{GM}{r^2} = a \mu\left(\frac{a}{a_0}\right)\tag{3}$$

At large distance ‘r’ at the galaxy outskirts, the kinematical acceleration ‘a’ is extremely-small smaller than $10^{-10}m/s^2$, i.e., $a \ll a_0$, hence the function $\mu\left(\frac{a}{a_0}\right) = \frac{a}{a_0}$; using this in Eq. (3),

$$\frac{GM}{r^2} = a \left(\frac{a}{a_0}\right)\tag{4}$$

which yields,

$$a = \left(\frac{GMa_0}{r^2}\right)^{\frac{1}{2}}\tag{5}$$

Also, the equation that relates to the centripetal-acceleration ‘a’ of a star orbiting in a circular orbit of radius ‘r’ with a velocity ‘v’ in the galaxy is

$$a = \frac{v^2}{r}.\tag{6}$$

Eqs. (5) and (6) lead to

$$v = (GMa_0)^{\frac{1}{4}}\tag{7}$$

Consequently, the velocity of star on circular orbit from the galaxy-center is constant and does not depend on the distance r; the rotational-curve is ‘flat’. The relationship $v = (GMa_0)^{\frac{1}{4}}$ between the flat rotational-velocity v to the observed mass M of the galaxy matches with observed flat velocity v to luminosity L (known as Tully-Fisher relation).

It may be noted that the critical acceleration, requiring MOND-correction, $a_0 \approx 10^{-10}m/s^2$ is negligibly small a value, that is why we never felt a need for modification in kinematical-acceleration as mostly on earth the accelerations ‘a’ are much higher than the critical value ‘ a_0 ’, hence $\mu(\frac{a}{a_0}) = 1$, thus $a_{effective} = a$. The effect of MOND is only noticeable to the centripetal-acceleration of galaxy’s rotational-arms wherein the acceleration $a \ll a_0$. MOND theory has been

quite successful [19-21], in explaining the galaxy-cluster-rotation and cosmology-behavior, specially the ‘flat’ curve of galaxy-arm’s rotational-velocity, without any need of otherwise-necessary dark-matter. The only drawback of MOND is that, it does not seem to have a good theoretical backing. Though a few μ -functions have been proposed in literature [19], e.g., one such is as following; but the theoretical-backing still lacks unless some theoretical-support (such as suggested in the present paper) is provided.

$$\mu\left(\frac{a}{a_0}\right) = \frac{\frac{a}{a_0}}{\left[1 + \left(\frac{a}{a_0}\right)^2\right]^{\frac{1}{2}}}, \quad (8)$$

which turn the function

$$\begin{aligned} \mu\left(\frac{a}{a_0}\right) &= 1 && \text{for } a \gg a_0, \\ &= \frac{a}{a_0} && \text{for } a \ll a_0. \end{aligned} \quad (9)$$

It is not that no work is done on theoretical aspects of MOND, even Milgrom [22] himself has talked about it and discussed at-length therein [22]. But the common sentiment (that still persists) is expressed occasionally that MOND (successful it may be) is only a ‘hypothesis’ that ‘saves the phenomena’, and that one day the origin of MOND-phenomenology may be found. Our present theoretical-attempt is with this spirit.

4 A Simple Theoretical Variant of MOND

The simple key for explaining the low-acceleration-limit MOND (which eliminates the need of dark-matter) lies in the dark-energy (which is responsible for the acceleration of the expanding universe, however, the acceleration is extremely-small as estimated in section 4.1).

4.1 Acceleration due to dark energy

Initially it was thought that the universe would be decelerating due to gravity inside, but now it is well established from several clues [4-12] such as Supernovae observation that the universe is actually accelerating due to repulsive-gravity of dark-energy. The deceleration-parameter q is defined as follows in Eq. (10). (Note that even though universe is actually accelerating, but the old-name deceleration-parameter retained; but q comes out to be actually negative, implying that \ddot{S} is positive i.e., universe accelerating). Note that though Scale-factor S (ratio of co-moving distance at previous-time at $Z > 0$ to the co-moving distance at present-time $Z = 0$) is dimensionless whereas Size-of-universe (or co-moving-distance) has dimension of length; but *sometimes* all these are denoted by the same symbol S , mainly in view that the scale-factor is proportional to the size-of-universe (co-moving-distance) and incidentally both the words begin

with the letter S. Sometimes, as in reference [2], scale-factor and universe-size are denoted by symbol ‘a’ (but we can not use such symbol here because in this paper ‘a’ is used for acceleration). So it is better for clarity, if all these are denoted by different symbols as follows; scale-factor as S, co-moving distance between two points as D, universe size as D_{max} , all being function of time due to expansion of universe. Thus scale factor $S(t) = \frac{D(t)}{D_0}$ or simply $S = \frac{D}{D_0}$ where D is the co-moving distance in the past (at $Z > 0$) and D_0 is the co-moving distance at present (at $Z = 0$). Hence it is obvious that S is proportional to D. Universe-size is D_{max} i.e., the distance of the visible universe-horizon, such that as per Hubble’s law $V_{max} = c = HD_{max}$. (Note that even if universe-tip may be moving with speed higher than light-speed, as it was during inflation, the observable ‘visible’ horizon will be limited by the equation $c = HD_{max}$. But in Eq. (11) what should we use for S ? It seems for galaxy observations, more appropriately S should be (being proportional) the co-moving-distance D, say, between the observed galaxy (say, Andromeda galaxy) from the earth (situated in the Milky-way galaxy).

The deceleration parameter q is defined by

$$q = -\frac{S\ddot{S}}{\dot{S}^2}. \quad (10)$$

Putting the experimental (See the Ref. [23]) value of $q = -0.67$, the expression for the acceleration (a_d) of the universe due to dark-energy is given by,

$$a_d = \ddot{S} = 0.67 \times \left(\frac{\dot{S}^2}{S} \right). \quad (11)$$

Note that from Hubble’s law of the expansion, velocity is proportional to distance i.e., $V = HD$, H being Hubble’s constant. Hubble’s law $V = HD$ is rewritten as $\dot{D} = HD$. This also gives the acceleration $\ddot{D} = H\dot{D} = \left(\frac{\dot{D}}{D}\right)\dot{D} = \frac{\dot{D}^2}{D}$ which is (almost) exactly of the same-form (equivalent) as the Eq. (11). Thus, this also reinforces the understanding that the co-moving distance D is proportional to scale-factor or vice-verse. This also indicates an important possibility that the Hubble’s expansion is due to dark-energy. Therefore, from (11), we derive, by replacing S by D and \dot{S} by \dot{D}

$$\begin{aligned} a_d &= 0.67 \times \frac{\dot{D}^2}{D} = 0.67 \times \frac{H^2 D^2}{D}, \\ &= 0.67 \times H^2 D = 0.67 \times HDH, \\ &= 0.67 \times H \left(\frac{D}{D_{max}} \right) D_{max} H = 0.67 \times H \left(\frac{D}{D_{max}} \right) c, \\ &= 0.67 \times cH \left(\frac{D}{D_{max}} \right), \end{aligned} \quad (12)$$

since it is known from Hubble’s law that maximum velocity of our visible universe-‘horizon’ can not exceed velocity of light (except during the inflation

time), hence in limiting case $H \times D_{max} = c$, where c is the speed of light.

Briefly,

$$a_d = K a_0 \approx 0.67 \times 6 \beta a_0 \quad (a_0 \approx cH \approx 1.2 \times 10^{-10} m/s^2). \quad (13)$$

It is better to express (as Eq. 13 than Eq. 12), for generality & accuracy, $a_d = K a_0$; where $K = 0.67 \times 6 \beta$ wherein the factor 6 comes-in because the value of $a_0 = 1.2 \times 10^{-10} \approx cH$ as suggested by MOND-proponent M. Milogram, but the actual value of $cH = 6.8 \times 10^{-10}$ which is about 6 times higher than a_0 . If the symbol D is taken as co-moving-distance of a galaxy, say for example, at 14 million light-years away which is thousand times less than the visible universe-size D_{max} (14 billion light-years); the distance ratio-factor $\beta = \frac{D}{D_{max}} = \frac{1}{1000}$. Hence, in that case of consideration, the value of K could be as $K = \frac{0.67 \times 6}{1000} = 0.004$. However, meanwhile, for qualitative theoretical explanation of the new variant of MOND, Eq. (13) would be used for subsequent analysis in the present paper.

4.2 The net effective gravitational (Newtonian) acceleration a_n

Since the universe's expansion is already accelerating with an acceleration of a_d ; the net effective 'kinetic' gravitational (Newtonian) acceleration ' a_n ' due to attractive force ($F = \frac{GMm}{r^2}$), will be equal to the modified 'effective kinematical-acceleration' which now equals to the central centripetal acceleration 'a *minus* the universe/galaxy's slow acceleration a_d ', i.e., $a_{effective} = a - a_d$. For equilibrium,

$$\begin{aligned} a_n &= a_{effective} = a - a_d, \\ &= a - K a_0, \\ &= a \left(1 - K \frac{a_0}{a} \right). \end{aligned} \quad (14)$$

The theoretical μ -function obtained here is simply $(1 - K \frac{a_0}{a})$, which becomes 1 for normal and large value of a , and is less than 1 if the value of a is extremely-small ($a \ll a_0$).

4.3 Galaxy's rotational velocity estimate

Since gravitational (Newtonian) acceleration $a_n = \frac{GM}{r^2}$ and that centripetal acceleration toward centre is known as $a = \frac{v^2}{r}$, the previous equation (14) reduces to

$$v = \left(\frac{GM}{r} + K a_0 r \right)^{\frac{1}{2}}. \quad (15)$$

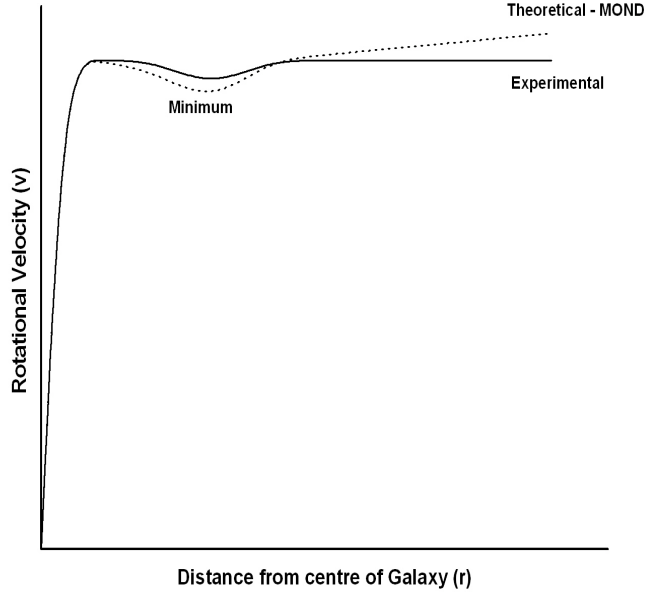


Figure 3: Rotational velocity versus distance from Galaxy-centre: experimental ‘flat’ curve with a Dip and a more-or-less flat theoretical curve with a minimum

For minimum v (or v^2), $\frac{dv^2}{dr} = 0$, gives that minimum velocity will occur at a certain value r^* as follows; this explains why there is a little dip in the generally-considered ‘flat’ rotational-velocity curve.

$$r^* = \left(\frac{GM}{K a_0} \right)^{\frac{1}{2}}. \quad (16)$$

The success of the present approach lies in the fact that not only the Eq. (15) gives more or less a ‘flat’ curve (due to decrease in v due to the first term $\frac{GM}{r}$ and increase in v due to the second term $K a_0 r$), but also predicts a dip (minimum) of this so-called ‘flat’ curve (Fig.3); note that these two terms together has a square-root sign (1/2 power) over it, trying to make the curve flatter after the dip. The explanation and agreement is good qualitatively, but detailed quantitative analysis is needed to be done.

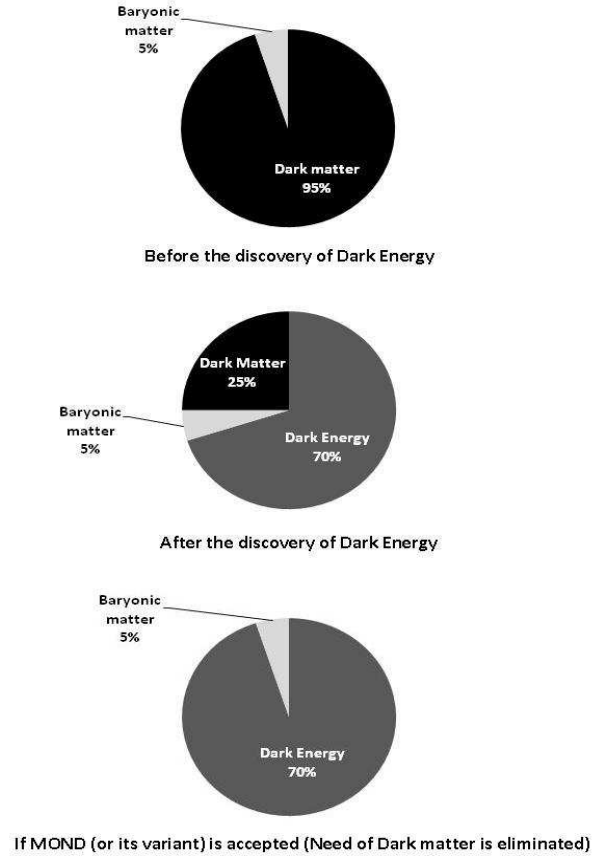


Figure 4: Possible Dark-constituents slices of the Universe

5 Re-slicing of the Constituent(s) of the Universe

Before the discovery of dark-energy, till 1998, it was thought that the dark-constituent (as shown in Fig.4 by shaded area) was having a single big slice of 95% dark-matter with at the most a 5% tiny slice of baryonic matter (stars, planets, galaxies, black-hole, white dwarf etc.). But after the discovery of dark-energy; the bigger dark-constituent slice was subdivided into two dark slices: smaller dark slice (about) 25% of dark-matter and the bigger dark slice (about) 70% of dark-energy. However, if MOND model which works well and eliminates the need of of dark-matter all-together, is accepted; then the composition of the dark-constituent(s) will be different, all the 95% would be dark-energy, as no dark-matter needed with MOND.

Both the dark matter and the dark energy are suggestions. But since, the acceleration (though small) of universe is now well established, the dark-energy seems more realistic than before. Dark-matter is rather quite strange and wizard; several schools of thoughts (cold and hot dark-matter and its candidates) have been proposed with no success as yet, as if the dark-matter is nothing but castle-in-air. In this situation; if MOND (which is an alternative to dark-matter theory) is accepted (specially after having a theoretical backing), the need of dark-matter is eliminated completely, making the dark slice of dark-energy alone. This may however, require a little modification (fine tuning) of the dark-energy theory.

6 Conclusions

Though there exists a successful empirical MOND theory, as proposed by the Israel-scientist M. Milgrom, for explaining the anomalous rotational-velocity of galaxies and beyond, but its drawback is that it lacks theoretical backing. In the present paper a novel variant of MOND is proposed on theoretical footing; this too at-present is able to explain the rotational-velocity curve qualitatively fairly well. MOND (or its variant) eliminates the need of dark-matter, the theoretical basis for the MOND-variant proposed herein lies in the acceleration of universe caused by dark-energy; meaning-by that it is the presence of dark-energy that eliminates the need of dark-matter. Though the present theory also predicts a dip in the so-called ‘flat’ curve in-agreement with the experimental-curve, more work on quantitative level needs to be done to establish the proposed-theory firmly. The present work gives a clear theoretical understanding of the so-far empirical MOND and starts a new beginning in this direction. The secret of high velocity depicted in flat rotational-velocity curve of big galaxy lies in the universe’s small acceleration (Eq. 13), caused by the mighty dark-energy possibly having its genesis [2] in the tiny nucleus.

Acknowledgements

The authors are thankful to Sushant Gupta for critical study of this research-paper. One of the authors (R. C. Gupta) is thankful to Project-students (S. Goyal, A. Kumar, I. Ansari & R. Singh) of IET Lucknow, and GLA, Mathura; and the other author (A. Pradhan) is thankful to IMSc., Chennai (Madras) for providing facility under associateship scheme; the places where the research-work is initiated and completed.

References

- [1] Beiser, A., Concept of Modern Physics, 6th Ed., Mc Graw Hill, New York (2003).
- [2] Gupta, R. C. and Pradhan, A., Int. J. Theor. Phys. **49**, 821 (2010).
- [3] Kaku, M., Beyond Einstein, Anchor Books, New York (1995).
- [4] Perlmutter, S., et al., Nature **391**, 51 (1998).
- [5] Perlmutter, S., et al., Astrophysics J. **517**, 565 (1999).
- [6] Riess, A. G., et al., Astron. J. **116**, 1009 (1998).
- [7] Riess, A. G., et al., Astron. J. **607**, 665 (2004).
- [8] Tonry, J. L., et al., Astrophys. J. **594**, 1 (2004).
- [9] Garnavich, P., et al., Astrophys. J. Lett. **493**, L53 (1998).
- [10] Schmidt, B. P., et al., Astrophys. J. **507**, 46 (1998).
- [11] Bahcall, N. A., et al., Science **284**, 1481 (1991).
- [12] Spergel, D.N., et al., Astrophys. J. Suppl. **148**, 175 (2003).
- [13] Zwicky, F., Helvetica Physica Acta **6**, 110 (1933).
- [14] Zwicky, F., Astrophys. J. **86**, 217 (1937).
- [15] Rubin, V. and Ford, W. K. Jr, Astrophys. **159**, 379 (1970).
- [16] Rubin, V., Thonnard, N. and Ford, W. K. Jr, Astrophys. J. **238**, 471 (1980).
- [17] Milgrom, M., Astrophys. J. **270**, 365 (1983).
- [18] Milgrom, M., Astrophys. J. **270**, 371 (1983).
- [19] Scarpa, R., arXiv:0601478 (2006).
- [20] Sanders, R. H., astro-ph/9710335 (1997).
- [21] Bekenstein, J. D., astro-ph/0701848 (2007).
- [22] Milgrom, M., J. New Astronomy Reviews **46**, 741 (2002), (arXiv: astro-ph/0207231v2).
- [23] Freedman, W. L. and Turner, S. S., Rev. Mod. Phys. **75**, 1433 (2003).